Chapter 6
Summary

Abstract  We give a short summary of the book ideas.

The problem of premixed flame propagation is studied as an example of unstable fronts that wrinkle on many scales. The analytic tool of pole expansion in the complex plane is employed to address the interaction of the unstable growth process with random initial conditions and perturbations. We argue that the effect of random noise is immense and that it can never be neglected in sufficiently large systems. We present simulations that lead to scaling laws for the velocity and acceleration of the front as a function of the system size and the level of noise, and analytic arguments that explain these results in terms of the noisy pole dynamics.

We consider premixed flame front propagation in channel geometries. The steady state solution in this problem is space dependent, and therefore the linear stability analysis is described by a partial integro-differential equation with a space dependent coefficient. Accordingly it involves complicated eigenfunctions. We show that the analysis can be performed to required detail using a finite order dynamical system in terms of the dynamics of singularities in the complex plane, yielding detailed understanding of the physics of the eigenfunctions and eigenvalues.

The roughening of expanding premixed flame fronts by the accretion of cusp-like singularities is a fascinating example of the interplay between instability, noise and nonlinear dynamics that is reminiscent of self-fractalization in Laplacian growth patterns. The nonlinear integro-differential equation that describes the dynamics of expanding premixed flame fronts is amenable to analytic investigations using pole decomposition. This powerful technique allows the development of a satisfactory understanding of the qualitative and some quantitative aspects of the complex geometry that develops in expanding premixed flame fronts.

Premixed flame propagation is used as a prototypical example of expanding fronts that wrinkle without limit in radial geometries but reach a simple shape in channel geometry. We show that the relevant scaling laws that govern the radial growth can be inferred once the simpler channel geometry is understood in detail. In radial geometries (in contrast to channel geometries) the effect of external noise is crucial in accelerating and wrinkling the fronts. Nevertheless, once the interrelations between system size, velocity of propagation and noise level are understood in channel geometry, the scaling laws for radial growth follow.
The mathematical problem of Laplacian growth without surface tension exhibits a family of exact (analytic) solutions in terms of logarithmic poles in the complex plane. We show that this family of solutions has a remarkable property: generic initial conditions in channel geometry which begin with arbitrarily many features exhibit an inverse cascade into a single finger. Moreover, it is possible to demonstrate that this solution is statistically stable: the width of the final finger will be oscillate near 1/2 of the channel width in the presence of a noise.